DEFINIZIONE ASSIOMATICA DI PROBABILITA'

$\Omega=\{\omega_i\}$

ALGEBRA DI EVENTI

$\Omega\in A$
$\forall A\in A, A^C\in A$
$\forall A, B\in A, A\cup B\in A$

$\Omega\in A\Rightarrow\Omega^C=\emptyset\in A$
$\forall A, B\in A\Rightarrow A^C, B^C\in A,$
$\Rightarrow A\cap B=\left(A^C\cup B^C\right)^C\in A$

σ - ALGEBRA DI EVENTI

$\Omega\in A$
$\forall A\in A, A^C\in A$
$\forall\{A_i\}\subseteq A, \bigcup_{i=1}^{\infty} A_i\in A$

$\Omega\in A\Rightarrow\Omega^C=\emptyset\in A$
$\{A_i\}\subseteq A\Rightarrow\{A_i^C\}\subseteq A,$
$\bigcup_{i=1}^{\infty} A_i^C\in A\Rightarrow\left(\bigcup_{i=1}^{\infty} A_i^C\right)^C=\bigcap_{i=1}^{\infty} A_i\in A$

DEFINIZIONE ASSIOMATICA DI PROBABILITA'

$P: A\rightarrow[0,1]$

$\forall A\in A, P(A)\geq 0$
$P(\Omega)=1$
$P(A\cup B)=P(A)+P(B)\forall A, B \quad A\cap B=\emptyset$
$P\left(\bigcup_{i=1}^{\infty} A_i\right)=\sum_{i=1}^{\infty} P(A_i) \quad \forall\{A_i\}, A_i\cap A_j=\emptyset\ \forall i=j$

$\Omega=A\cup A^C, A\cap A^C=\emptyset$
$1=P(\Omega)=P(A\cup A^C)=P(A)+P(A^C)$
$P(A^C)=1-P(A)$

$\emptyset=\Omega^C$
$P(\emptyset)=1-P(\Omega)=0$

TEOREMA DELLA PROBABILITA' TOTALE

Dato $(\Omega, A, P) \quad \forall A, B\in A$
$P(A\cup B)=P(A)+P(B)-P(A\cap B)$
se A e B sono disgiunti
$A\cap B=\emptyset, P(A\cap B)=0 \quad P(A\cup B)=P(A)+P(B)$
se A e B non sono disgiunti
$A\cup B=(A-B)\cup B \quad P(A\cup B)=P(A-B)+P(B)$
$A=(A-B)\cup(A\cap B) \quad P(A)=P(A-B)+P(A\cap B) \quad P(A-B)=P(A)-P(A\cap B)$
$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

CASO DISCRETO E FINITO

$A=\bigcup_{i=1}^{n}\omega_i \quad P(A)=\sum_{i=1}^{n} P(\omega_i)$

$\forall i, \quad P(\omega_i)=\frac{1}{N} P(A)=\sum_{i=1}^{n} P(\omega_i)=\frac{|A|}{|\Omega|}=\frac{n}{N}$

PROBABILITA' CONDIZIONATA

$P(A|B)=\frac{P(A\cap B)}{P(B)}$ Dato B , $P(B)>0$, $P(\bullet|B)$ soddisfa le proprietà

1 $P(A|B)=\frac{P(A\cap B)}{P(B)}\geq 0,\forall A\in \boldsymbol{A}$

2 $P(\Omega|B)=\frac{P(\Omega\cap B)}{P(B)}=1$

3 $A_1\subset A_2\Rightarrow P(A_1|B)\leq P(A_2|B)$ $A_{1,}A_2\in \boldsymbol{A}$ $A_1\subset A_2\Rightarrow A_1 B\subset A_2 B$

$P(A_1|B)=\frac{P(A_1\cap B)}{P(B)}\leq\frac{P(A_2\cap B)}{P(B)}=P(A_2|B)$

4 se $A_{1,}A_2\in \boldsymbol{A}$ e $A_1\cap A_2=\emptyset$ allora $P(A_1\cup A_2|B)=P(A_1|B)+P(A_2|B)$

$P(A_1\cup A_2|B)=\frac{P((A_1\cup A_2)\cap B)}{P(B)}=\frac{P((A_1\cap B)\cup(A_2\cap B))}{P(B)}=\frac{P(A_1\cap B)\cup P(A_2\cap B)}{P(B)}$

TEOREMI:

con $P(B)>0$

$P(\emptyset|B)=0$ $P(A|B)+P(A^C|B)=1$

$P(A_1|B)=P(A_1\cap A_2|B)+P(A_1\cap A_2^C|B)$ $A_{1,}A_{2,}B\in \boldsymbol{A}$

$P(A_1\cup A_2|B)=P(A_1|B)+P(A_2|B)-P(A_1\cap A_2|B)$ $A_{1,}A_{2,}B\in \boldsymbol{A}$

$P\left(\bigcup_{i=1}^{n} A_i|B\right)\leq\sum_{i=1}^{n} P(A_i|B)$ $A_1,\ldots,A_n,B\in \boldsymbol{A}$

INDIPENDENZA STOCASTICA

$P(A|B)=P(A)$ $P(A\cap B)=P(A)P(B)$

$P(A|B)=\frac{P(A\cap B)}{P(B)}=P(A)\Rightarrow P(A\cap B)=P(A)P(B)$

$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)P(B)}{P(B)}\Rightarrow P(A|B)=P(A)$

se A_1,A_2,A_3 sono indipendenti $\Rightarrow P(A_1\cap A_2\cap A_3)=P(A_1)P(A_2)P(A_3)$

$A_1,A_2,A_3,\ldots,A_n$ sono indipendenti se $P\left(\bigcap_{i=1}^{n} A_i\right)=\prod_{i=1}^{n} P(A_i)$

LEGGE DI BAYES

dati $(\Omega,\boldsymbol{A},P(\bullet)),B_1,\ldots,B_n\in \boldsymbol{A}$, $P(B_i)>0,$ $B_i\cap B_j=\emptyset,i\neq j,$ $\bigcup_{i=1}^{n} B_i=\Omega$

allora $\forall A\in \boldsymbol{A}$ vale: $P(B_K|A)=\frac{P(A|B_K)P(B_K)}{\sum_{i=1}^{n} P(A|B_i)P(B_i)}$

$P(B|A)=\frac{P(A|B)P(B)}{P(A)}=\frac{P(A|B)P(B)}{P(A|B)P(B)+P(A|B^C)P(B^C)}$

TEOREMA DELLE PROBABILITA' TOTALI II

dati $(\Omega,\boldsymbol{A},P(\bullet)),B_1,\ldots,B_n\in \boldsymbol{A}$, $\forall i$, $P(B_i)>0,$ $B_i\cap B_j=\emptyset,i\neq j,$ $\bigcup_{i=1}^{n} B_i=\Omega$

allora $\forall A\in \boldsymbol{A}$ vale: $P(A)=\sum_{i=1}^{n} P(A|B_i)P(B_i)$